Propagation of spherical shock waves in stars

By AKIRA SAKURAI*
Department of Applied Mathematics,
Weizmann Institute of Science, Rehovot, Israel

(*Received* 26 *April* 1956)

SUMMARY

Propagation of spherical shock waves through self-gravitating polytropic gas spheres such as stars, caused by an instantaneous central explosion of finite energy E, is discussed theoretically. The problem is characterized by two lengths R_0, L, where

$$R_0 = \left(\frac{E}{4\pi p_0}\right)^{1/3}, \qquad L = \left(\frac{3C_0^2}{2\pi\rho_0 G}\right)^{1/2},$$

p_0, ρ_0 and C_0 are the values of pressure, density and velocity of sound at the centre of the equilibrium pre-explosion state, and G is the constant of gravitation. R_0 and L are scales connected with the power of the explosion and the dimensions of the star respectively, and their ratio $A = R_0/L$ has a fundamental significance. A solution especially suitable in the case of $A = O(1)$ is developed in the form of power series in R/R_0 (R is the distance between the shock front and the centre) by a method similar to that used in previous papers by the present author (1953, 1954). An approximation to this solution is carried out up to the term in R^3. In particular, the velocity of the shock wave U is found to be

$$\frac{U}{C_0} = 1{\cdot}30\left(\frac{R}{R_0}\right)^{-3/2}\left\{1+0{\cdot}41A^2\left(\frac{R}{R_0}\right)^2+0{\cdot}57\left(\frac{R}{R_0}\right)^3+\ldots\right\}$$

for the case of $\gamma = 1{\cdot}4$, where γ is the ratio of specific heats.

1. INTRODUCTION

The purpose of the present paper is to investigate the propagation of spherical shock waves through gravitating gas spheres, such as stars, caused by an instantaneous central explosion of finite energy. The problem has previously been treated by Kopal (1954) and Sedov (1954) for the case of constant shock front strength in a special distribution of density in the pre-explosion state. The case of variable shock strength is discussed by Lidov (1955) and Rogers (1956). We consider the problem more generally, with variable shock strength and the pre-explosion state simply given by the equilibrium equation of self-gravitating polytropic gas spheres. In the case of uniform pre-explosion states, the author (1953, 1954) discussed the problem by a method of constructing the solution in power series of U^{-2},

* Weizmann Fellow, on leave from Tokyo Electrical Engineering College.

where U is the propagation velocity of the shock front. A similar method is also applicable to the present problem, if we use R (the distance between the shock front and the centre) in place of U^{-2}. We cannot now use U^{-2} as an independent variable as was done in the previous papers, because U is no longer necessarily a monotonically decreasing function of increasing R, but may even increase as a result of decreasing pressure, density, etc. in the equilibrium state, and would then cease to be single-valued. An approximation to this type of solution is carried out up to the term in R^3. The terms in R^0 and R^3 can be found from the results of the previous papers, since they contain no effect of non-uniformity in the pre-explosion state. Further, the term in R vanishes automatically, and the effect of non-uniform pre-explosion distribution comes only from the term in R^2 in this approximation. Since $U^{-2} = O(R^3)$, this just corresponds to the second approximation in the previous paper (1954) including terms up to U^{-2}.

The present problem is characterized by two lengths R_0, L, where

$$R_0 = \left(\frac{E}{4\pi p_0}\right)^{1/3}, \tag{1}$$

$$L = \left(\frac{3C_0^2}{2\pi\rho_0 G}\right)^{1/2}, \tag{2}$$

and E is the energy of explosion, p_0, ρ_0 and C_0 are values of pressure, density and velocity of sound at the centre in the equilibrium pre-explosion state, and G is the constant of gravitation. R_0 represents a scale connected with the effective range of the power of the explosion and appears also in the previous papers (1953, 1954). L represents, on the other hand, a length connected with the dimensions of the star. The ratio A of these two lengths defined as

$$A = R_0/L, \quad \text{or} \quad A^2 = \left(\frac{\pi}{54}\right)^{1/3} \rho_0 G C_0^{-2} p_0^{-2/3} E^{2/3}, \tag{3}$$

has a fundamental significance. The present theory is specially suitable for the case of $A = O(1)$, that is, the case in which the effective range of the explosion is of the same order of magnitude as the scale of the star. In the case of a weak explosion ($A \ll 1$), the theory is still valid but reduces to that for the case of the uniform pre-explosion state; it then seems to be more suitable to consider the problem by Whitham's method (1953). In the case of $A \gg 1$, we have a very strong explosion, far bigger than the star itself, and it is clear that this phenomenon requires completely different formulation.

We formulate in §2 the fundamental system of equations given by equations of motion, conservation of the total explosion energy and boundary conditions of the shock front. The solution in power series of R is developed in §3, and the term in R^2, which is the only new one required for the fourth approximation, is found by numerical integration in §4. In §5 and §6, miscellaneous results obtained from this fourth approximation to the solution are discussed.

2. Fundamental equations

The equations governing the spherically symmetrical flow of a polytropic gas of adiabatic index γ under the influence of its own gravitation are (Kopal & Lin 1951)

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{Gm}{r^2}, \tag{4}$$

$$\frac{D\rho}{Dt} = -\rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right), \tag{5}$$

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho, \tag{6}$$

$$\frac{D(p\rho^{-\gamma})}{Dt} = 0, \tag{7}$$

where $m(r, t)$, $u(r, t)$, $p(r, t)$ and $\rho(r, t)$ denote respectively the mass inside a sphere of radius r, the velocity, the pressure and the density at a distance r from the origin at time t, and the expression D/Dt denotes $D/Dt = \partial/\partial t + u\partial/\partial r$. By means of equation (5), equation (7) transforms into

$$\frac{Dp}{Dt} = -\gamma p\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right). \tag{8}$$

For the equilibrium state where $u = 0$, $\partial/\partial t = 0$, we put

$$p = p'(R), \qquad \rho = \rho'(R), \qquad m = m'(R), \qquad r = R; \tag{9}$$

thus equations (4), (6) and (7) are reduced to

$$\frac{1}{\rho'}\frac{dp'}{dR} + \frac{Gm'}{R^2} = 0, \qquad \frac{dm'}{dR} = 4\pi R^2\rho', \qquad p'\rho'^{-\gamma} = p_0\rho_0^{-\gamma}, \tag{10}$$

while equations (5) and (8) give merely the relations $\rho = \rho'$ and $p = p'$ in this case.

For the radius of the shock front it will be convenient to use the same symbol R as above. This R is a function of t and connected with the propagation velocity at the front U by the relation

$$dR/dt = U. \tag{11}$$

At the shock front ($r = R$), we have the following Rankine–Hugoniot conditions:

$$\left.\begin{aligned} \frac{u}{U} &= \frac{2}{\gamma+1}\left\{1-\left(\frac{C}{U}\right)^2\right\}, \qquad \frac{p}{p'} = \frac{2\gamma}{\gamma+1}\left(\frac{U}{C}\right)^2 - \frac{\gamma-1}{\gamma+1}, \\ \frac{\rho}{\rho'} &= \frac{\gamma+1}{\gamma-1}\left\{1+\frac{2}{\gamma-1}\left(\frac{C}{U}\right)^2\right\}^{-1}, \qquad m = m', \end{aligned}\right\} \tag{12}$$

where C is the sound velocity in the equilibrium state and is given by

$$C = \left(\frac{\gamma p'}{\rho'}\right)^{1/2}. \tag{13}$$

Since we consider the case of an instantaneous explosion of energy E, the equation of energy may be written

$$\int_0^R\left(\frac{1}{2}u^2+\frac{1}{\gamma-1}\frac{p}{\rho}-\frac{Gm}{r}\right)\rho 4\pi r^2\,dr-\int_0^R\left(\frac{1}{\gamma-1}\frac{p'}{\rho'}-\frac{Gm'}{R}\right)\rho' 4\pi R^2\,dR = E. \tag{14}$$

In place of r, t, we now introduce new independent variables x, z defined by

$$\frac{r}{R} = x, \qquad \frac{R}{R_0} = z, \tag{15}$$

and express the quantities u, p, ρ, m; p', ρ', m' as follows:

$$\frac{u}{U} = f(x, z), \quad \frac{p}{p_0} = P\left(\frac{U}{C}\right)^2 g(x, z), \quad \frac{\rho}{\rho_0} = Dh(x, z), \quad \frac{m}{m_0} = Mi(x, z), \tag{16}$$

$$\frac{p'}{p_0} = P(z), \quad \frac{\rho'}{\rho_0} = D(z), \quad \frac{m'}{m_0} = M(z), \quad m_0 = \tfrac{4}{3}\pi\rho_0 R_0^3, \tag{17}$$

where f, g, h, i; P, D, M are non-dimensional new variables. Using (15), (16) and (17), we have

$$\frac{\partial}{\partial r} = \frac{1}{R}\frac{\partial}{\partial x}, \qquad \frac{D}{Dt} = \frac{U}{R}\left\{(f-x)\frac{\partial}{\partial x}+z\frac{\partial}{\partial z}\right\}. \tag{18}$$

We now substitute (15), (16), (17) and (18) into the fundamental equations (4), (5), (6), (8), (10), (12) and (14). Then (4), (5), (6) and (8) become

$$(f-x)\frac{\partial f}{\partial x}+z\frac{\partial f}{\partial z}+\frac{z}{U}\frac{dU}{dz}f = -\frac{1}{\gamma h}\frac{\partial g}{\partial x}-2A^2\frac{C_0^2}{U^2}\frac{M}{z}\frac{i}{x^2}, \tag{19}$$

$$(f-x)\frac{1}{h}\frac{\partial h}{\partial x}+\frac{z}{h}\frac{\partial h}{\partial z}+\frac{z}{D}\frac{dD}{dz} = -\frac{\partial f}{\partial x}-\frac{2f}{x}, \tag{20}$$

$$\frac{\partial i}{\partial x} = 3z^3\frac{D}{M}x^2h, \tag{21}$$

$$(f-x)\frac{1}{g}\frac{\partial g}{\partial x}+\frac{z}{g}\frac{\partial g}{\partial z}+\frac{z}{D}\frac{dD}{dz}+\frac{2z}{U}\frac{dU}{dz} = -\gamma\left(\frac{\partial f}{\partial x}+\frac{2f}{x}\right), \tag{22}$$

while equations (10) become

$$\frac{1}{D}\frac{dP}{dz}+\frac{2\gamma A^2}{z^2}M = 0, \qquad \frac{dM}{dz} = 3z^2D, \qquad PD^{-\gamma} = 1, \tag{23}$$

where we have used (1), (2) and (3). The boundary conditions (12) become

$$f(1,z) = \frac{2}{\gamma+1}\left\{1-\left(\frac{C}{U}\right)^2\right\}, \qquad g(1,z) = \frac{2\gamma}{\gamma+1}-\frac{\gamma-1}{\gamma+1}\left(\frac{C}{U}\right)^2,$$

$$h(1,z) = \frac{\gamma+1}{\gamma-1}\left\{1+\frac{2}{\gamma-1}\left(\frac{C}{U}\right)^2\right\}^{-1}, \qquad i(1,z) = 1, \tag{24}$$

while the energy equation (14) gives

$$\frac{U^2}{C_0^2}Dz^3\int_0^1\left(\frac{\gamma}{2}hf^2+\frac{g}{\gamma-1}\right)x^2\,dx-2\gamma A^2MDz^2\int_0^1 hix\,dx-$$
$$-\int_0^z\left(\frac{P}{\gamma-1}-2\gamma A^2\frac{MD}{z}\right)z^2\,dz = 1.$$

If we write $\int_0^1 \left(\frac{\gamma}{2} hf^2 + \frac{g}{\gamma - 1}\right) x^2 \, dx = J, \qquad \int_0^1 hix \, dx = K,$

$$\int_0^z \left(\frac{P}{\gamma - 1} - 2\gamma A^2 \frac{MD}{z}\right) z^2 \, dz = I,$$

our equation is reduced to

$$\left.\begin{aligned} &\frac{U^2}{C_0^2} Dz^3 J - 2\gamma A^2 MDz^2 K - I = 1, \\ \text{or} \quad &\frac{C_0^2}{U^2} = z^3 JD(1 + I + 2\gamma A^2 MDz^2 K)^{-1}. \end{aligned}\right\} \tag{25}$$

3. The solution in power series of z

Following the method used in the earlier paper (1953), we construct the solution in power series of z. At first we find the solution for the equilibrium state expressed by (23). Eliminating M and P from (23), we have

$$\frac{d}{dz}\left(z^2 D^{\gamma - 2} \frac{dD}{dz}\right) + 6A^2 z^2 D = 0, \tag{26}$$

from which we get

$$D = 1 - A^2 z^2 + \frac{13 - 5\gamma}{10} A^4 z^4 + \ldots; \tag{27}$$

and then, from (23) and (13), P, M and C are given by

$$P = 1 - \gamma A^2 z^2 + \tfrac{4}{5}\gamma A^4 z^4 + \ldots, \tag{28}$$

$$M = z^3 \left\{1 - \tfrac{3}{5} A^2 z^2 + \frac{39 - 15\gamma}{70} A^4 z^4 + \ldots\right\}, \tag{29}$$

$$\frac{C^2}{C_0^2} = 1 - (\gamma - 1) A^2 z^2 + \tfrac{3}{10}(\gamma - 1) A^4 z^4 + \ldots. \tag{30}$$

In figure 1, the expression (27) for the case $\gamma = 1{\cdot}4$ is compared with the exact solution of (26) which was given by Eddington (1930).

We now assume

$$\left.\begin{aligned} f(x, z) &= f_0 + zf_1 + z^2 f_2 + z^3 f_3 + \ldots, \\ g(x, z) &= g_0 + zg_1 + z^2 g_2 + z^3 g_3 + \ldots, \\ h(x, z) &= h_0 + zh_1 + z^2 h_2 + z^3 h_3 + \ldots, \\ i(x, z) &= i_0 + zi_1 + z^2 i_2 + z^3 i_3 + \ldots, \end{aligned}\right\} \tag{31}$$

where $f_\nu(x)$, $g_\nu(x)$, $h_\nu(x)$, $i_\nu(x)$ ($\nu = 0, 1, 2, \ldots$) are unknown functions to be determined.

We insert expressions (27), (28), (29), (30) and (31) in equation (25), and then obtain

$$J = J_0(1 + z\alpha_1 + z^2\alpha_2 + z^3\alpha_3 + \ldots), \tag{32}$$

where

$$
\left.
\begin{aligned}
J_0 &= \int_0^1 \left(\frac{\gamma}{2} h_0 f_0^2 + \frac{g_0}{\gamma - 1}\right) x^2 \, dx, \\
\alpha_1 J_0 &= \int_0^1 \left(\frac{\gamma}{2} h_1 f_0^2 + \gamma f_0 h_0 f_1 + \frac{g_1}{\gamma - 1}\right) x^2 \, dx, \\
\alpha_2 J_0 &= \int_0^1 \left\{\frac{\gamma}{2} h_2 f_0^2 + \gamma f_0 h_0 f_2 + \frac{\gamma}{2} f_1 (h_0 f_1 + 2 f_0 h_1) + \frac{g_2}{\gamma - 1}\right\} x^2 \, dx, \\
\alpha_3 J_0 &= \int_0^1 \left\{\frac{\gamma}{2} h_3 f_0^2 + \gamma f_0 h_0 f_3 + \frac{\gamma}{2} f_1 (h_1 f_1 + 2 f_0 h_2) + \right. \\
&\quad \left. + \gamma (f_0 f_2 h_1 + h_0 f_1 f_2) + \frac{g_3}{\gamma - 1}\right\} x^2 \, dx,
\end{aligned}
\right\} \tag{33}
$$

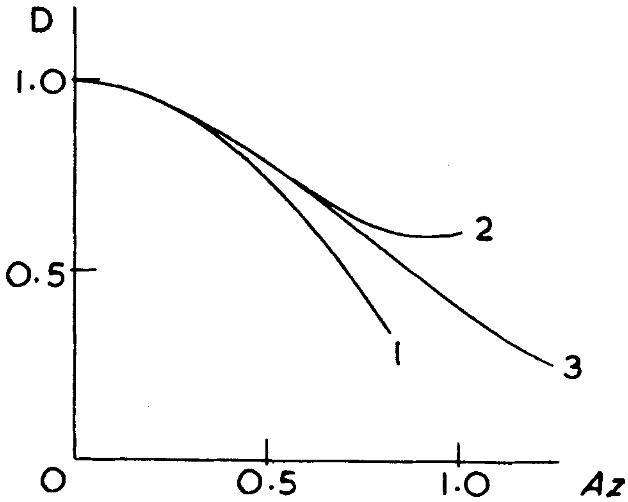

Figure 1. Comparison of approximate density distributions for $\gamma = 1{\cdot}4$ with exact one. Curve 1, $1-(Az)^2$; curve 2, $1-(Az)^2+0{\cdot}6(Az)^4$; curve 3, exact solution.

and
$$K = K_0(1 + z\beta_1 + z^2\beta_2 + z^3\beta_3 + \dots), \tag{34}$$

where

$$
\left.
\begin{aligned}
K_0 &= \int_0^1 h_0 i_0 x \, dx, \\
\beta_1 K_0 &= \int_0^1 (h_1 i_0 + h_0 i_1) x \, dx, \\
\beta_2 K_0 &= \int_0^1 (h_2 i_0 + h_0 i_2 + h_1 i_1) x \, dx,
\end{aligned}
\right\} \tag{35}
$$

and
$$MD = z^3(1-\tfrac{8}{5}A^2z^2+\dots), \tag{36}$$

$$I = \frac{z^3}{\gamma-1}\left\{\frac{1}{3}-\frac{2\gamma-1}{5}\gamma A^2z^2+\frac{4\gamma}{35}(4\gamma-3)A^4z^4+\dots\right\}, \tag{37}$$

Using (32), (33), (34), (35), (36) and (37), we finally get from (25)

$$\frac{C_0^2}{U^2} = J_0 z^3\left\{1+\alpha_1 z+(\alpha_2-A^2)z^2+\left(\alpha_3-\frac{1}{3}\frac{1}{\gamma-1}-\alpha_1 A^2\right)z^3+\dots\right\}, \tag{38}$$

and hence

$$\begin{aligned}\frac{z}{U}\frac{dU}{dz} &= -\tfrac{3}{2}-\tfrac{1}{2}\alpha_1 z-(\alpha_2-A^2-\tfrac{1}{2}\alpha_1^2)z^2-\left\{\frac{3}{2}\left(\alpha_3-\frac{1}{3}\frac{1}{\gamma-1}\right)+\right.\\ &\qquad\left.+\tfrac{1}{2}\alpha_1(\alpha_1^2-3\alpha_2)\right\}z^3+\dots,\\ &= -\tfrac{3}{2}(1+\delta_1 z+\delta_2 z^2+\delta_3 z^3+\dots),\end{aligned}$$

where

$$\delta_1 = \tfrac{1}{3}\alpha_1,\qquad \delta_2 = \tfrac{2}{3}(\alpha_2-A^2-\tfrac{1}{2}\alpha_1^2),\qquad \delta_3 = \alpha_3-\frac{1}{3}\frac{1}{\gamma-1}+\tfrac{1}{3}\alpha_1(\alpha_1^2-3\alpha_2). \tag{39}$$

Equation (38) gives the relation between the propagation velocity U and the position $R = R_0 z$ of the shock front, if we know the values of constants J_0, α_1, α_2, ... for a given A.

Now, substituting (27), (28), (29), (30), (31), (38) and (39) into (19), (20), (21) and (22), and comparing the coefficients of the same powers of z on both sides of (19), (20), (21) and (22), we get the following system of equations:

for the term independant of z

$$\left.\begin{aligned}(f_0-x)f_0'-\tfrac{3}{2}f_0 &= -\frac{g_0'}{\gamma h_0},\\ (f_0-x)\frac{h_0'}{h_0} &= -f_0'-\frac{2}{x}f_0,\\ (f_0-x)\frac{g_0'}{g_0} &= 3-\frac{2\gamma}{x}f_0-\gamma f_0',\\ i_0' &= 3x^2h_0;\end{aligned}\right\} \tag{40}$$

for the first power of z

$$\left.\begin{aligned}(f_0-x)f_1'+\frac{1}{\gamma h_0}g_1' &= (\tfrac{1}{2}-f_0')f_1+\frac{g_0'}{\gamma h_0^2}h_1+\tfrac{3}{2}f_0\,\delta_1,\\ (f_0-x)\left(\frac{h_1}{h_0}\right)'+f_1' &= -\left(\frac{h_0'}{h_0}+\frac{2}{x}\right)f_1-\frac{h_1}{h_0},\\ (f_0-x)\left(\frac{g_1}{g_0}\right)'+\gamma f' &= -\left(\frac{g_0'}{g_0}+\frac{2\gamma}{x}\right)f_1-\frac{g_1}{g_0}+3\delta_1,\\ i_1' &= 3x^2h_1;\end{aligned}\right\} \tag{41}$$

for the second power of z

$$
\left.\begin{aligned}
(f_0-x)f_2'+\frac{1}{\gamma h_0}g_2' &= -(\tfrac{1}{2}+f_0)f_2+\frac{g_0'}{\gamma h_0^2}h_2+\tfrac{3}{2}f_0\,\delta_2+\\
&\quad +f_1(-f_1'+\tfrac{3}{2}\delta_1)+\frac{h_1}{\gamma h_0^2}\left(g_1'-g_0'\frac{h_1}{h_0}\right),\\
(f_0-x)\left(\frac{h_2}{h_0}\right)'+f_2' &= -\left(\frac{h_0'}{h_0}+\frac{2}{x}\right)f_2-2\frac{h_2}{h_0}+2A^2+\\
&\quad +(f_0-x)\frac{h_1}{h_0}\left(\frac{h_1}{h_0}\right)'-\left(\frac{h_1}{h_0}\right)'f_1+\left(\frac{h_1}{h_0}\right)^2,\\
(f_0-x)\left(\frac{g_2}{g_0}\right)'+\gamma f_2' &= -\left(\frac{g_0'}{g_0}+\frac{2\gamma}{x}\right)f_2-2\frac{g_2}{g_0}+2A^2+3\delta_2+\\
&\quad +(f_0-x)\frac{g_1}{g_0}\left(\frac{g_1}{g_0}\right)'-\left(\frac{g_1}{g_0}\right)'f_1+\left(\frac{g_1}{g_0}\right)^2,\\
i_2' &= 3x^2(h_2-\tfrac{2}{5}A^2h_0);
\end{aligned}\right\}\qquad(42)
$$

for the third power of z

$$
\left.\begin{aligned}
(f_0-x)f_3'+\frac{1}{\gamma h_0}g_3' &= -(\tfrac{3}{2}+f_0')f_3+\frac{g_0'}{\gamma h_0^2}h_3+\tfrac{3}{2}\delta_3 f_0+\\
&\quad +f_2(-f_1'+\tfrac{3}{2}\delta_1)+f_1(-f_2'+\tfrac{3}{2}\delta_2)+\frac{h_1}{\gamma h_0^2}\left(g_2'-g_1'\frac{h_1}{h_0}+g_0'\frac{h_1^2}{h_0^2}\right)+\\
&\quad +\frac{h_2}{\gamma h_0^2}\left(g_1'-2g_0'\frac{h_1}{h_0}\right),\\
(f_0-x)\left(\frac{h_3}{h_0}\right)'+f_3' &= -\left(\frac{h_0'}{h_0}+\frac{2}{x}\right)f_3-3\frac{h_3}{h_0}+\\
&\quad +(f_0-x)\left\{\frac{h_1}{h_0}\left(\frac{h_2}{h_0}\right)'-\left(\frac{h_1^2}{h_0^2}-\frac{h_2}{h_0}\right)\left(\frac{h_1}{h_0}\right)'\right\}-f_1\left\{\left(\frac{h_2}{h_0}\right)'-\frac{h_1}{h_0}\left(\frac{h_1}{h_0}\right)'\right\}-\\
&\quad -f_2\left(\frac{h_1}{h_0}\right)'-\frac{h_1}{h_0}\left(\frac{h_1^2}{h_0^2}-3\frac{h_2}{h_0}\right),\\
(f_0-x)\left(\frac{g_3}{g_0}\right)+\gamma f_3' &= -\left(\frac{g_0'}{g_0}+\frac{2\gamma}{x}\right)f_3-3\frac{g_3}{g_0}+3\delta_3+\\
&\quad +(f_0-x)\left\{\frac{g_1}{g_0}\left(\frac{g_2}{g_0}\right)'-\left(\frac{g_1^2}{g_0^2}-\frac{g_2}{g_0}\right)\left(\frac{g_1}{g_0}\right)'\right\}-f_1\left(\frac{g_2}{g_0}\right)'-\frac{g_1}{g_0}\left(\frac{g_1}{g_0}\right)'\Big\}-\\
&\quad -f_2\left(\frac{g_1}{g_0}\right)'-\frac{g_1}{g_0}\left(\frac{g_1^2}{g_0^2}-3\frac{g_2}{g_0}\right),\\
i_3' &= 3x^2\left(h_3-\frac{2}{5}A^2h_1\right);
\end{aligned}\right\}\qquad(43)
$$

where the primes (′) denote differentations with respect to x.

From the boundary conditions (24), we have in a similar manner

$$
f_0(1)=\frac{2}{\gamma+1},\qquad g_0(1)=\frac{2\gamma}{\gamma+1},\qquad h_0(1)=\frac{\gamma+1}{\gamma-1},\qquad i_0(1)=1,\qquad(44)
$$

$$f_1(1) = 0, \qquad g_1(1) = 0, \qquad h_1(1) = 0, \qquad i_1(1) = 0, \tag{45}$$

$$f_2(1) = 0, \qquad g_2(1) = 0, \qquad h_2(1) = 0, \qquad i_2(1) = 0, \tag{46}$$

$$f_3(1) = -\frac{2}{\gamma+1}J_0, \quad g_3(1) = -\frac{\gamma-1}{\gamma+1}J_0, \quad h_3(1) = -\frac{2(\gamma+1)}{(\gamma-1)^2}J_0, \quad i_3(1) = 0. \tag{47}$$

The first step in the solution of the problem is to solve the system of differential equations (40) about $f_0(x)$, $g_0(x)$, $h_0(x)$, $i_0(x)$ with the boundary conditions (44), and to insert this solution in the first equation of (33). Then we have the first approximation to the solution in the form

$$u = Uf_0(x), \quad p = p_0\left(\frac{U}{C_0}\right)^2 g_0(x), \quad \rho = \rho_0 h_0(x), \quad m = m_0 i_0(x),$$
$$\left(\frac{C_0}{U}\right)^2 = J_0 z^3. \tag{48}$$

Since i_0 enters only in the fourth equation in (40), we can find f_0, g_0, h_0 separately from the first three equations. It is clear that this procedure is precisely the same as that of the first approximation in the case of a uniform pre-explosion state, which was first developed by Taylor (1950). The solutions $f_0(x)$, $g_0(x)$, $h_0(x)$, J_0 were, in the previous paper (1953), written as $f^{(0)}(x)$, $g^{(0)}(x)$, $h^{(0)}(x)$, J_0. The value of J_0 for $\gamma = 1{\cdot}4$ is $0{\cdot}596$, and some values of f_0, g_0 and h_0 are shown in table 1.

The second step is to find the functions $f_1(x)$, $g_1(x)$, $h_1(x)$, $i_1(x)$ and a constant δ_1. For this purpose we use the system of differential equations (41), with the boundary conditions (45) and the integral condition (the second equation of (33)) with the relation $\delta_1 = \frac{1}{3}\alpha_1$ from (39). The equations (41) and the integral condition are, however, linear and homogeneous, about f_1, g_1, h_1, i_1, δ_1, and so we have simply

$$f_1(x) \equiv 0, \quad g_1(x) \equiv 0, \quad h_1(x) \equiv 0, \quad i_1(x) \equiv 0, \quad \delta_1 = \alpha_1 \equiv 0. \tag{49}$$

Equations (49) lead to a simplification of the equation for the third step, and we get from (42) and (46)

$$\left.\begin{aligned}
(f_0-x)f_2' + \frac{1}{\gamma h_0}g_2' &= -\left(\frac{1}{2}+f_0'\right)f_2 + \frac{g_0'}{\gamma h_0^2}h_2 + \frac{3}{2}f_0\,\delta_2,\\
(f_0-x)\left(\frac{h_2}{h_0}\right)' + f_2' &= -\left(\frac{h_0'}{h_0}+\frac{2}{x}\right)f_2 - 2\frac{h_2}{h_0} + 2A^2,\\
(f_0-x)\left(\frac{g_2}{g_0}\right)' + \gamma f_2' &= -\left(\frac{g_0'}{g_0}+\frac{2\gamma}{x}\right)f_2 - 2\frac{g_2}{g_0} + 2A^2 + 3\delta_2,\\
i_2' &= 3x^2\left(h_2 - \frac{2}{5}A^2h_0\right),\\
f_2(1) = g_2(1) &= h_2(1) = i_2(1) = 0,
\end{aligned}\right\} \tag{50}$$

and from (33) and (39)

$$\alpha_2 J_0 = \int_0^1 \left(\tfrac{1}{2}\gamma h_2 f_0^2 + \gamma f_0 h_0 f_2 + \frac{1}{\gamma-1} g_2\right) x^2\, dx, \tag{51}$$

$$\delta_2 = \tfrac{2}{3}(\alpha_2 - A^2). \tag{52}$$

By means of (50) and (52), we can find the functions $f_2(x)$, $g_2(x)$, $h_2(x)$, $i_2(x)$ and the constants α_2, δ_2. The solution is given below in § 4.

Equations for the fourth step are also simplified by means of (49), and we get from (43) and (47)

$$\left.\begin{aligned}
(f_0 - x)f_3' + \frac{1}{\gamma h_0} g_3' &= -\left(\frac{3}{2} + f_0'\right) f_3 + \frac{g_0'}{\gamma h_0^2} h_3 + \frac{3}{2}\delta_3 f_0,\\
(f_0 - x)\left(\frac{h_3}{h_0}\right)' + f_3' &= -\left(\frac{h_0'}{h_0} + \frac{2}{x}\right) f_3 - 3\frac{h_3}{h_0},\\
(f_0 - x)\left(\frac{g_3}{g_0}\right)' + \gamma f_3' &= -\left(\frac{g_0'}{g_0} + \frac{2\gamma}{x}\right) f_3 - 3\frac{g_3}{g_0} + 3\delta_3,\\
i_3' &= 3x^2 h_3,\\
f_3(1) = -\frac{2}{\gamma+1} J_0, \quad g_3(1) &= -\frac{\gamma-1}{\gamma+1} J_0, \quad h_3(1) = -\frac{2(\gamma+1)}{(\gamma-1)^2} J_0,\\
i_3(1) &= 0,
\end{aligned}\right\} \tag{53}$$

and from (33) and (39)

$$\left.\begin{aligned}
\alpha_3 J_0 &= \int_0^1 \left(\frac{\gamma}{2} h_3 f_0^2 + \gamma h_0 f_0 f_3 + \frac{1}{\gamma-1} g_3\right) x^2\, dx,\\
\delta_3 &= \alpha_3 - \frac{1}{3}\frac{1}{\gamma-1}.
\end{aligned}\right\} \tag{54}$$

Now we put

$$f_3 = J_0(x - f_0)\phi_3, \quad g_3 = J_0 g_0 \psi_3, \quad h_3 = J_0 h_0 \chi_3, \quad \delta_3 = J_0 \lambda_3. \tag{55}$$

Then we get

$$\left.\begin{aligned}
-(x - f_0)\phi_3' + \frac{g_0}{\gamma h_0 (x - f_0)}\psi_3' &= -(2f_0' + \tfrac{1}{2})\phi_3 + \left(f_0' + \frac{3}{2}\frac{f_0}{x - f_0}\right) \times\\
&\quad \times (\chi_3 - \psi_3) + \frac{3}{2}\frac{f_0}{x - f_0}\lambda_3,\\
(x - f_0)(-\phi_3' + \chi_3') &= 3(\phi_3 + \chi_3),\\
(x - f_0)(-\gamma\phi_3' + \psi_3') &= 3\{(\gamma-1)\phi_3 + \psi_3 - \lambda_3\},\\
i_3' &= 3J_0 x^2 h_0 \chi_3,\\
\phi_3(1) = -\frac{2}{\gamma-1},\ \psi_3(1) = -\frac{\gamma-1}{2\gamma},\ &\chi_3(1) = -\frac{2}{\gamma-1},\ i_3(1) = 0,\\
\int_0^1 \left\{\gamma f_0 h_0 (x - f_0)\phi_3 + \frac{g_0}{\gamma-1}\psi_3 + \frac{\gamma}{2} f_0^2 h_0 \chi_3\right\} x^2\, dx &= \lambda_3 J_0 + \frac{1}{3}\frac{1}{\gamma-1},
\end{aligned}\right\} \tag{56}$$

where we have used equations (40) for f_0, g_0, h_0.

By comparing this system of equations (56) for ϕ_3, ψ_3, χ_3, i_3, λ_3 with that for $\phi, \psi, \chi, \lambda$, expressed by (2), (3), (4), (5) and (6) in the previous paper (1954), we can find that in the case of $\alpha = 2$ (spherical wave) they are precisely the same except for the equation for i_3. Then we get

$$\phi_3(x) \equiv \phi(x), \qquad \psi_3(x) \equiv \psi(x), \qquad \chi_3(x) \equiv \chi(x), \qquad \lambda_3 \equiv \lambda_1. \tag{57}$$

The value of λ_3 for $\gamma = 1{\cdot}4$ is $-1{\cdot}918$, and some values of ϕ_3, ψ_3, χ_3 are also given in table 1.

x	$f_0(x)$	$g_0(x)$	$h_0(x)$	$\phi_3(x)$	$\psi_3(x)$	$\chi_3(x)$
1·00	0·833	1·167	6·000	−5·000	−0·143	−5·000
0·95	0·751	0·760	2·464	−3·941	0·779	0·879
0·90	0·685	0·595	1·234	−3·603	0·475	2·567
0·80	0·584	0·471	0·392	−3·45	−0·18	3·32
0·60	0·429	0·427	0·041	—	−0·51	—
0·00	0·000	0·426	0·000	−3·5	−0·53	3·5

Table 1

We have now arrived at the fourth approximation to the solution, including the terms up to z^3. Since $U^{-2} \propto z^3(1+\ldots)$ by (38), this fourth approximation corresponds to the second approximation of the previous paper (1954); because, in the former case, the solution was expanded in powers of $y = (C/U)^2$, where C denoted the velocity of sound in the uniform atmosphere. The fact that we can express the coefficient of z^3 in the present solution in terms of the coefficient of y in the previous result is due to this reason. But it must be remarked that the errors involved in the two approximations are not the same, since the error in the present case is $O(z^4)$, while in the previous case it is $O(y^2)$, i.e. $O(z^6)$. Thus, to have the same error, we must proceed to the sixth approximation. The non-uniform pre-explosion state, caused by gravitation and expressed by the constant A, does not affect the terms in z^0 and z^3, but appears separately in the term of z^2 in the fourth approximation. In the case of $A = 0$, equations (50) and (52) become homogeneous, and the term in z^2 vanishes as in the case of the term in z; the solution then reduces to that in the case of a uniform pre-explosion state.

4. Solutions for f_2, g_2, h_2, δ_2

We now solve equations (50) and (52) to the third approximation. The procedure is similar to that of the second approximation (the term in y) developed in the previous paper (1954). We put

$$f_2 = A^2(x-f_0)\phi_2, \qquad g_2 = A^2 g_0\,\psi_2, \qquad h_2 = A^2 h_0\,\chi_2, \qquad \alpha_2 = A^2\sigma_2, \tag{58}$$

so that (50) and (52) are reduced to

$$-(x-f_0)\phi_2' + \frac{g_0}{\gamma h_0(x-f_0)}\psi_2' = (\tfrac{1}{2}-2f_0')\phi_2 + \frac{g_0'}{\gamma h_0(x-f_0)}(\chi_2-\psi_2) + \frac{f_0}{x-f_0}(\sigma_2-1), \tag{59}$$

$$(x-f_0)(-\phi_2'+\chi_2') = 3\phi_2+2\chi_2-2, \tag{60}$$

$$(x-f_0)(-\gamma\phi_2'+\psi_2') = 3(\gamma-1)\phi_2+2\psi_2-2\sigma_2, \tag{61}$$

$$i_2' = 3A^2h_0\,x^2(\chi_2-\tfrac{2}{5}), \tag{62}$$

$$\phi_2(1) = \psi_2(1) = \chi_2(1) = i_2(1) = 0, \tag{63}$$

$$\left.\begin{aligned} \sigma_2 J_0 &= \int_0^1\left\{\gamma f_0\,h_0(x-f_0)\phi_2+\frac{g_0}{\gamma-1}\psi_2+\frac{\gamma}{2}f_0^2\,h_0\,\chi_2\right\}x^2\,dx, \\ \delta_2 &= \tfrac{3}{2}A^2(\delta_2-1), \end{aligned}\right\} \tag{64}$$

where we have used (40). Since i_2 appears only in (62), we can exclude it. The problem is then to solve the system of differential equations (59), (60) and (61) for ϕ_2, ψ_2, χ_2 with the boundary conditions (63), and to put this solution, which includes σ_2, into equation (64) and determine σ_2. It is remarkable that A has no role in this process.

Just as in the previous papers (1953, 1954), we have an intermediate integral from equations (60) and (61). Let us perform the operation $(5\gamma-3)\times(60)-5\times(61)$; we have

$$3\phi_2'-5\psi_2'+(5\gamma-3)\chi_2' = 2\{3\phi_2-5\psi_2+(5\gamma-3)\chi_2+5\sigma_2-5\gamma+3\}.$$

Integrating this and inserting the condition (63) to determine the integration constant, we obtain the relation

$$3\phi_2-5\psi_2+(5\gamma-3)\chi_2+5\sigma_2-5\gamma+3 = (5\sigma_2-5\gamma+3)\exp\left(\int_1^x\frac{2\,dx}{x-f_0}\right). \tag{65}$$

Eliminating ϕ_2' from (59) and (61), we have

$$\psi_2' = a\phi_2+b\psi_2+c\chi_2+e\sigma_2+d, \tag{66}$$

where

$$a = -\left(2f_0'-\tfrac{1}{2}+3\frac{\gamma-1}{\gamma}\right)f, \qquad d = -\frac{f_0}{x-f_0}f,$$

$$b = -\left(f_0'+\frac{3}{2}\frac{f_0}{x-f_0}+\frac{2}{\gamma}\right)f, \qquad e = \left(\frac{f_0}{x-f_0}+\frac{2}{\gamma}\right)f,$$

$$c = \left(f_0'+\frac{3}{2}\frac{f_0}{x-f_0}\right)f, \qquad f = \gamma\frac{x-f_0}{g_0}h_0\left\{1-\frac{(x-f_0)^2}{g_0}h_0\right\}^{-1}.$$

Now we split up ϕ_2, ψ_2, χ_2 into two parts

$$\phi_2 = \phi_{21}+\sigma_2\phi_{22}, \qquad \psi_2 = \psi_{21}+\sigma_2\psi_{22}, \qquad \chi_2 = \chi_{21}+\sigma_2\chi_{22}, \tag{67}$$

and substitute these in (66), (61), (65), (63), (64). The equations split up into two independent systems for ϕ_{21}, ψ_{21}, χ_{21} and ϕ_{22}, ψ_{22}, χ_{22} respectively, viz.

$$\left.\begin{aligned} \psi_{21}' &= a\phi_{21}+b\psi_{21}+c\chi_{21}+d, \\ \phi_{21}' &= \frac{1}{\gamma}\left[\psi_{21}'-\frac{1}{x-f_0}\{3(\gamma-1)\phi_{21}+2\psi_{21}\}\right], \\ \chi_{21} &= \frac{-1}{5\gamma-3}(3\phi_{21}-5\psi_{21})+1-\exp\left(\int_1^x\frac{2\,dx}{x-f_0}\right), \\ \text{with}\quad \phi_{21}(1) &= \psi_{21}(1) = \chi_{21}(1) = 0, \end{aligned}\right\} \tag{68}$$

and
$$\left.\begin{aligned}\psi'_{22} &= a\phi_{22}+b\psi_{22}+c\chi_{22}+e,\\ \phi'_{22} &= \frac{1}{\gamma}\left[\psi'_{22}-\frac{1}{x-f_0}\{3(\gamma-1)\phi_{22}+2\psi_{22}-2\}\right],\\ \chi_{22} &= \frac{1}{5\gamma-3}\left\{5\exp\left(\int_1^x\frac{2\,dx}{x-f_0}\right)-5-3\phi_{22}+5\psi_{22}\right\},\end{aligned}\right\} \tag{69}$$
with
$$\phi_{22}(1) = \psi_{22}(1) = \chi_{22}(1) = 0.$$

Moreover
$$\sigma_2 J_0 = J_1+\sigma_2 J_2, \quad \text{or} \quad \sigma_2 = J_1/(J_0-J_2), \tag{70}$$
where
$$J_1 = \int_0^1\left\{\gamma f_0\,h_0(x-f_0)\phi_{21}+\frac{g_0}{\gamma-1}\psi_{21}+\frac{\gamma}{2}f_0^2\,h_0\,\chi_{21}\right\}x^2\,dx,$$
$$J_2 = \int_0^1\left\{\gamma f_0\,h_0(x-f_0)\phi_{22}+\frac{g_0}{\gamma-1}\psi_{22}+\frac{\gamma}{2}f_0^2\,h_0\,\chi_{22}\right\}x^2\,dx.$$

For $\gamma = 1{\cdot}4$, (68) and (69) were integrated numerically from $x = 1$ with the initial values $\phi_{21} = \psi_{21} = \chi_{21} = \phi_{22} = \psi_{22} = \chi_{22} = 0$. From $x = 1$ to $x = 0{\cdot}96$, the Runge–Kutta method was used, taking the steps of the numerical integration as 0·02. In the remaining part of the range, the Levy method was applied, again with a step of 0·02. Inserting these values of ϕ_{21}, ψ_{21}, χ_{21}; ϕ_{22}, ψ_{22}, χ_{22} into (70), we get the value

$$\sigma_2 = 0{\cdot}182, \quad \text{with } J_0 = 0{\cdot}596, \quad \lambda_3 = -1{\cdot}918. \tag{71}$$

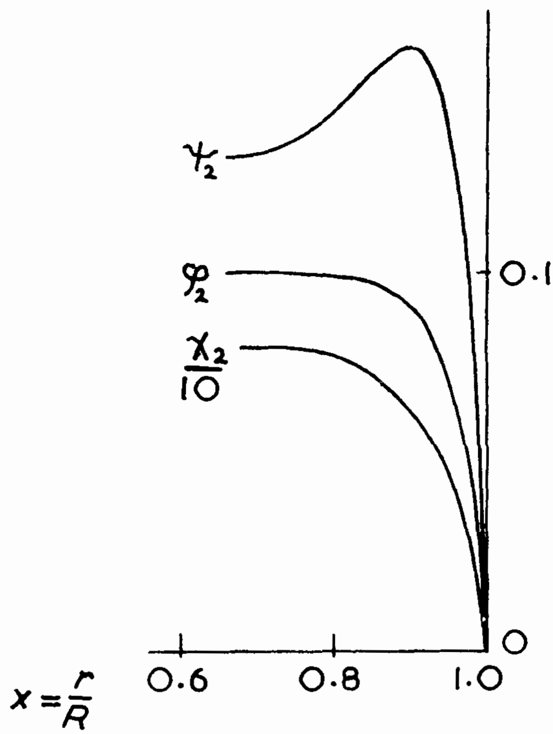

Figure 2. The solutions ϕ_2, ψ_2, χ_2 for $\gamma = 1{\cdot}4$.

Using this value of σ_2, ϕ_2, ψ_2, χ_2 were determined finally by (67); they are tabulated in table 2, and are shown in figure 2.

x	$\phi_2(x)$	$\psi_2(x)$	$\chi_2(x)$
1·00	0·000	0·000	0·000
0·98	0·037	0·081	0·232
0·96	0·061	0·124	0·386
0·94	0·076	0·146	0·495
0·92	0·086	0·157	0·572
0·90	0·091	0·159	0·629
0·88	0·094	0·158	0·671
0·86	0·097	0·155	0·704
0·84	0·097	0·151	0·730
0·82	0·098	0·147	0·752
0·80	0·098	0·143	0·764
0·78	0·10	0·14	0·78
0·76	0·10	0·14	0·80
0·74	0·10	0·14	0·81
0·72		0·13	0·81
0·70		0·13	0·82
0·68		0·13	0·82
0·66			0·82

Table 2

5. Velocity–distance curves and time–distance curves

Inserting (49), (55), (57) and (58) in equation (38), we get

$$\left(\frac{C_0}{U}\right)^2 = J_0\, z^3\{1+(\sigma_2-1)A^2z^2+J_0\,\lambda_3\, z^3+\ldots\}.$$

By use of the values J_0, σ_2, λ_3 given in (71), and with R/R_0 in place of z, this equation becomes, for $\gamma = 1{\cdot}4$,

$$\left(\frac{C_0}{U}\right)^2 = 0{\cdot}596\left(\frac{R}{R_0}\right)^3\left\{1-0{\cdot}82A^2\left(\frac{R}{R_0}\right)^2-1{\cdot}14\left(\frac{R}{R_0}\right)^3\ldots\right\}$$

or

$$\frac{U}{C_0} = 1{\cdot}30\left(\frac{R}{R_0}\right)^{-3/2}\left\{1+0{\cdot}41A^2\left(\frac{R}{R_0}\right)^2+0{\cdot}57\left(\frac{R}{R_0}\right)^3\ldots\right\}. \quad (72)$$

Velocity–distance curves (U–R curves) given by (72) are shown in figure 3 for $A^2 = 0$, 1, 2, 5 and 10.

We can see in figure 3 that A has the effect of preventing U from decreasing, and this effect increases with larger A as expected. It is interesting to consider U/C, the strength of the shock wave, which is obtained by multiplication of (72) and (30). In the case of $\gamma = 1{\cdot}4$, we have

$$\left(\frac{C}{U}\right)^2 = \left(\frac{C_0}{U}\right)^2\left(\frac{C}{C_0}\right)^2 = 0{\cdot}596\left(\frac{R}{R_0}\right)^3\left\{1-1{\cdot}22A^2\left(\frac{R}{R_0}\right)^2-1{\cdot}14\left(\frac{R}{R_0}\right)^3\ldots\right\},$$

or

$$\frac{U}{C} = 1{\cdot}30\left(\frac{R}{R_0}\right)^{-3/2}\left\{1+0{\cdot}61A^2\left(\frac{R}{R_0}\right)^2+0{\cdot}57\left(\frac{R}{R_0}\right)^3\ldots\right\}, \quad (73)$$

from which we can see that A is more effective in maintaining the strength of the shock wave.

Using the expressions (73) and (12), we can deduce relations between the radius of the shock front R and quantities such as pressure and density at the shock front. To obtain the most interesting of these relations, we consider the temperature T at the shock front; we have

$$\frac{T}{T'} = \frac{p}{p'}\frac{\rho'}{\rho} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}\left(\frac{U}{C}\right)^2\left\{1+\left(\frac{2}{\gamma-1}-\frac{\gamma-1}{2\gamma}\right)\left(\frac{C}{U}\right)^2+\ldots\right\},$$

where T' denotes the value in the equilibrium state. Since $T'/T_0 = (C/C_0)^2$, where T_0 denotes the value of T' at the centre, we have finally

$$\frac{T}{T_0} = 0{\cdot}326\left(\frac{R}{R_0}\right)^{-3}\left\{1+0{\cdot}82A^2\left(\frac{R}{R_0}\right)^2+4{\cdot}04\left(\frac{R}{R_0}\right)^3+\ldots\right\}. \qquad (74)$$

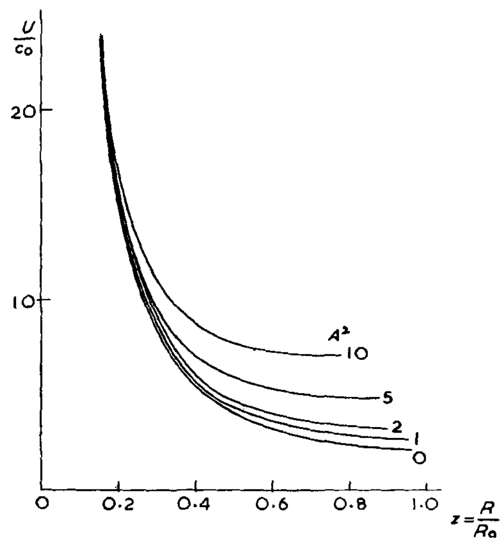

Figure 3. Velocity–distance curves for $A^2 = 0, 1, 2, 5$ and 10.

In this case, A does not have so much effect. The distance–time relation can be obtained from an integration of (72) with use of the relation (11). This gives

$$C_0\,dt = \frac{1}{1{\cdot}30}\left(\frac{R}{R_0}\right)^{3/2}\left\{1-0{\cdot}41A^2\left(\frac{R}{R_0}\right)^2-0{\cdot}57\left(\frac{R}{R_0}\right)^3\ldots\right\}dR$$

and we get

$$\frac{C_0\,t}{R_0} = 0{\cdot}308\left(\frac{R}{R_0}\right)^{5/2}\left\{1-0{\cdot}23A^2\left(\frac{R}{R_0}\right)^2-0{\cdot}27\left(\frac{R}{R_0}\right)^3\ldots\right\}. \qquad (75)$$

This equation is represented in figure 4 for $A^2 = 0, 1, 2, 5$ and 10.

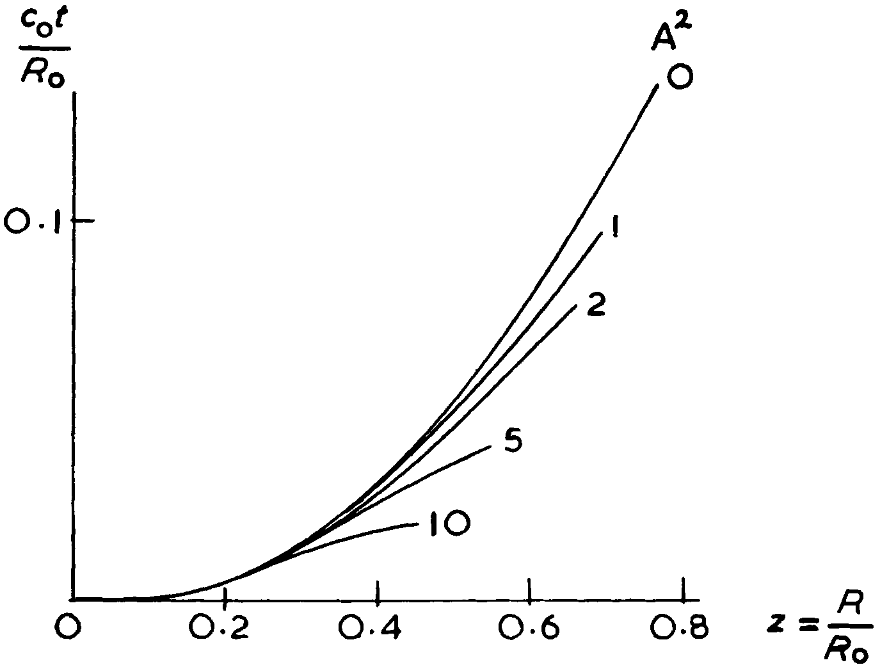

Figure 4. Time–distance curves for $A^2 = 0, 1, 2, 5$ and 10.

If we combine (75) and (74), we can get a relation between the temperature T and the time t.

6. Distribution of velocity, density and temperature

The velocity u, pressure p, density ρ and temperature θ behind the shock front are given by (16), (31), (27), (28), (72), (73), (49), (55) and (58), that is,

$$\left.\begin{aligned} \frac{u}{C_0} &= \frac{U}{C_0}\{f_0 + (x - f_0)(A^2 z^2 \phi_2 + 0{\cdot}596 z^3 \phi_3 \ldots)\}, \\ \frac{U}{C_0} &= 1{\cdot}30 z^{-3/2}(1 + 0{\cdot}41 A^2 z^2 + 0{\cdot}57 z^3 + \ldots), \end{aligned}\right\} \tag{76}$$

$$\left.\begin{aligned} \frac{p}{p_0} &= P\left(\frac{U}{C}\right)^2 g_0\{1 + A^2 z^2 \psi_2 + 0{\cdot}596 z^3 \psi_3 + \ldots\}, \\ P &= 1 - 1{\cdot}4 A^2 z^2 + \ldots, \\ \left(\frac{U}{C}\right)^2 &= 1{\cdot}68 z^{-3}\{1 + 1{\cdot}22 A^2 z^2 + 1{\cdot}14 z^3 + \ldots\}, \end{aligned}\right\} \tag{77}$$

$$\left.\begin{aligned} \frac{\rho}{\rho_0} &= D h_0\{1 + A^2 z^2 \chi_2 + 0{\cdot}596 z^3 \chi_3 + \ldots\}, \\ D &= 1 - A^2 z^2 \ldots, \end{aligned}\right\} \tag{78}$$

$$\frac{\theta}{T_0} = \frac{p}{p_0} \times \frac{\rho_0}{\rho}. \tag{79}$$

A special interest attaches to the variation of ρ and p with time, as shown in figures 5 and 6 for the case of $A^2 = 2$. In figure 5 distributions of ρ/ρ_0 when $z = R/R_0 = 0{\cdot}1$, $0{\cdot}3$, $0{\cdot}5$, which correspond to $C_0t/R_0 = 0{\cdot}001$, $0{\cdot}014$, $0{\cdot}046$ respectively, are shown with the initial pre-explosion distribution $D = \rho'/\rho_0 = 1-2z^2$. We can see that ρ/ρ_0 increases to several times the initial value D at the shock front, and decreases to zero toward the centre. This decrease is so rapid that there is always a region of almost vanishing density near the centre, and this region is also expanding with time. The same kind of distributions of z^3p/p_0 for the same values of z and the initial distribution $P = p'/p_0 = 1-2{\cdot}8z^2$ are shown in figure 6. In this case, the increase of p/p_0 from P at the shock front is very large, especially for small z,

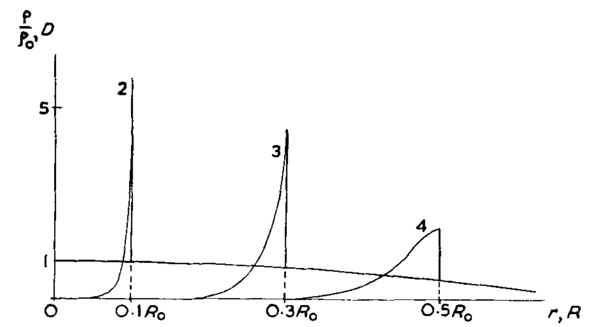

Figure 5. Distributions of density for $A^2 = 2$ at various stages. Curve 1, initial pre-explosion distribution D; curve 2, distribution of ρ/ρ_0 at $z = R/R_0 = 0{\cdot}1$; curve 3, distribution of ρ/ρ_0 at $z = R/R_0 = 0{\cdot}3$; curve 4, distribution of ρ/ρ_0 at $z = R/R_0 = 0{\cdot}5$.

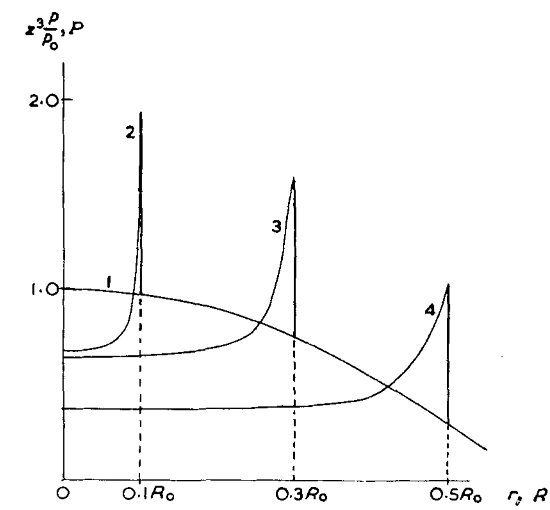

Figure 6. Distributions of pressure for $A^2 = 2$ at various stages. Curve 1, initial pre-explosion distribution P; curve 2, distribution of z^3p/p_0 at $z = R/R_0 = 0{\cdot}1$; curve 3, distribution of z^3p/p_0 at $z = R/R_0 = 0.3$; curve 4, distribution of z^3p/p_0 at $z = R/R_0 = 0{\cdot}5$.

and is about two thousand for $z = 0{\cdot}1$. The value of $z^2 p/p_0$ is shown in the figure, though P is depicted without change. Corresponding to the region of vanishingly small density, we now have the constant pressure region of about the same size. This value of constant pressure changes rapidly with z, being roughly proportional to z^{-3} or $(U/C)^{-2}$. The distribution of temperature, which can be obtained from (79), using the above values of p/p_0 and ρ/ρ_0, then has a region of almost infinite temperature corresponding to the above regions.

References

Eddington, A. S. 1930 *The Internal Constitution of Stars.* Cambridge University Press.

Kopal, Z. 1954 *Astrophys. J.* **120**, 159.

Kopal, Z. & Lin, C. C. 1951 *Proc. Nat. Acad. Sci., Wash.*, **37**, 495.

Lidov, M. L. 1955 *Prikl. Mat. Meh.* **19**, 541.

Rogers, M. H. 1956 *Proc. Roy. Soc.* A, **235**, 120.

Sakurai, A. 1953 *J. Phys. Soc. Japan* **8**, 662.

Sakurai, A. 1954 *J. Phys. Soc. Japan* **9**, 256.

Sedov, L. I. 1954 *Methods of Similarity and Dimensions in Mechanics.* Moscow: Gosudarstv. Izdat. Tehn-Teor. Lit.

Taylor, G. I. 1950 *Proc. Roy. Soc.* A, **201**, 159.

Whitham, G. B. 1953 *Comm. Pure Appl. Math.* **6**, 397.